The Child's Perspective on Place Value: Five Ways Children Conceptualize Two-Digit Numbers

Rob Schoen: Place value is a topic where it's so important to pay attention to those details in how students are thinking—what they're showing you, what words they're using, what gestures they're making—in order to get that information about what they do understand and remembering all the while that the child's perspective on this topic is so very different from yours.

Wendy Bray: In this module, we are going to tease apart the five different ways that children conceptualize two-digit numbers. This framework draws on the collective findings of multiple research projects and is reported in the 1997 article by Karen Fuson and colleagues in the *Journal for Research in Mathematics Education*. So children initially see 2-digit numbers in very much the same way that they see smaller numbers: as a collection of ones. Take, for example, Oliver and his way of determining the quantity of 35 cubes.

Wendy: *What I want you to do is just tell me how many cubes are there?*

Wendy: The first step to understanding the quantity 35 is being able to count one by one to 35 such that each object is counted exactly one time. Then, a child needs to understand that the last count—35—is the number that describes the quantity of the set.

Oliver: *35.*

Zak Champagne: I think we have some good evidence in that video that Oliver's definitely thinking about 35 as a group of 35 ones.

Rob: And Oliver clearly has some good strategies for counting. He knows the number words, and he's using one to one correspondence and separating the cubes he's counted from the ones he hasn't yet counted.

Wendy: So, next I asked Oliver to write down the number 35 on paper, and he was able to write the numeral 35—he had the connection that this set of cubes is 35. But then I asked him to tell me what the digit 5 and what the digit 3 meant and he, he just wasn't sure.

Wendy: *No? Okay. How about…does this part of the number have anything to do with how many cubes you have?*

Oliver: *No.*

Wendy: *No? You're not sure?*

Tanya Vik Blais: And this is really typical. He's a first grader, so when he sees a set of 35 ones and writes the number 35, the 3 and the 5 don't hold any separate meaning for him.

Wendy: Right, this set of cubes is 35 ones. It's a single set of 35 ones. So then at some point, children start to notice that in that number name, thirty-five, there's the thirty part, and there's the five part. And so, they start to notice that, in their numbers, that there's this part that can be thirty and this part that can be five. And maybe that has something to do with those digits that are in the number, 35.

Zak: So it is tricky for some kids to think about this three as 30, even as they're making the connection with the number word and they can tell you that the 3 represents 30, they sometimes struggle to associate that with 30 cubes in the set.

Tanya: Thinking about the times when you ask kids to write the number 35, and they write "305," you guys seen that? It's really common and we sometime grumble or have concerns when a kid writes 35 that way, but this might really have to do with what we're talking about: about beginning to recognize the 30 part separate from the 5 part in this number.

Wendy: Right, and they're really thinking about 35 as two sets of ones, as 30 ones and as 5 ones. And it's actually quite significant because that opens the doors to numbers being broken apart in other ways.

Zak: Like in tens.

Wendy: Right, exactly. As children have more experiences with two-digit numbers, they develop two distinct ways of thinking in tens to conceptualize quantities: counting by tens and recognizing units of ten. Becoming fluent with both ways of thinking in tens is essential to developing an integrated understanding. A child using counting-by-tens thinking organizes objects into sets of 10 and uses counting by tens to figure out how many.

Grace: *1, 2, 3, 4, 5. 1, 2, 3, 4, 5. 10, 20. 1, 2, 3, 4, 5…6, 7, 8, 9, 10. 30. Then, 31, 32, 33, 34, 35.*

Wendy: Notice when Grace counted 10, 20, 30, she seemed to be thinking about 10 ones and then counting another 10, then 20 ones, and then counting another 10, 30 ones. 10, 20, 30 but always ones.

Rob: Right, so each one is sort of each time she counts a group of 10, then there's 10 more, and 10 more, but it's not necessarily one 10, two 10s, three 10s.

Wendy: From the evidence in the video, she sees each set as 10 ones and she needs to use the counting sequence—10, 20, 30—in order to figure out how many.

Tanya: Sometimes kids are capable of counting by tens but they won't use that understanding unless they're presented with objects that are already grouped that way. When I interviewed

Anton, he counted by ones on the 35 cubes task, so next I gave him 26 loose cubes and some cups. I had him put 10 cubes in each of the two cups and left the remaining 6 cubes on the table. With the groups of ten already formed, he counted by tens.

Anton: *26.*

Tanya: *How did you count that?*

Anton: *I did 10, 20, then 21, 22, 23, 24, 25, 26.*

Tanya: *Very nice.*

Tanya: I think that Anton really shows us that part of thinking in tens is using that understanding in a situation where it would be really helpful, like in counting a large quantity of cubes.

Zak: So yeah, just because a child knows the counting-by-tens sequence doesn't mean he'll use it when he's counting a set of objects.

Wendy: So, the second way of thinking in tens is recognizing units of ten. So, with this approach, I not only know that this has ten ones, but I also know that it's one ten and I can count- one ten, two tens, three tens—this is three tens. And so a child simultaneously seeing those ten ones and also at the same time knowing that that's one of something. That it's one ten.

Tanya: I've been guilty in the past, in working with second graders, of when they're counting 1, 2, 3, I stop and actually correct them because I want to hear 10, 20, 30. So in some ways, I have actually unintentionally kind of, pulled them back from being able to use this recognizing units of ten when they count.

Rob: Interesting.

Wendy: Right, so what we want is for kids to see that those ten ones, but also one ten and also be able to think about it as 1, 2, 3, tens and also that this three tens is 10, 20, 30. So, you know, just because a child sees that 1, 2, 3 tens doesn't necessarily mean that the child knows that it's

30. They might need to then use that counting by tens—10, 20, 30—to realize that it's, that's the quantity.

Zak: It's these subtleties that are so interesting, right, between 1, 2, 3; or 1, 2, 3 tens; or 1, 2, 3, thirty. All of those things give us insight into maybe something different that a child is thinking.

Wendy: So, after children have developed these two thinking-in-tens conceptions, counting by tens—10, 20, 30—and recognizing units of ten—being able to think of this as 1 ten, 2 tens, 3 tens—I have 3vtens, the next step, where we want them to go now is being able to develop an integrated understanding. The kind that toggles back and forth between those ways of thinking and being able to do it pretty quickly. So, being able to think "I know that this is 30 because I see 3 tens, and 3 tens is 30. I don't have to go back and count 10, 20, 30 anymore."

Tanya: You know, and that's the really tricky part. You know, I see sometimes that students with some tasks seem to really understand place value and then on other tasks, they don't. So, it's really kind of this idea of now I see it and now I don't.

Zak: So, I had the pleasure of interviewing a second grader named Sofia, and she showed some great thinking in tens. She provided a really strong base-10 understanding, on this 35 cubes task.

Zak interviews Sofia

Sofia: *35.*

Zak: *35. And can you tell me how you got 35?*

Sofia: *Because there were separate tens so I counted one ten, and then I counted another ten, and then another 10, which was three, so it's 30, and then I counted the extras which was five.*

Zak: *I see just how you were thinking about that.*

Zak: So, after the counting task, I gave Sofia a word problem involving four groups of ten.

Zak: *Your teacher has 4 new boxes of markers. There are 10 markers in each box. How many new markers does she have?*

Sofia: *40.*

Zak: *And how did you get 40?*

Sofia: *Because there were 4 new boxes and there's 10 in each one, there would be 4 tens, and that would be 40.*

Zak: *So, four tens is the same as 40 to you? Is that right? I see.*

Rob: Notice that Sofia, she knew 40 right away. She didn't need to draw anything or write anything down; she just knew that it was 40. So, I gave her a different word problem.

Zak: *Your teacher has 30 pretzels. She has a big bag of pretzels in the room, and she's going to dump them all out. She wants to put the pretzels into snack bags so there are 10 pretzels in each bag. She's going to get some pretzels and put 10 in this bag, get some pretzels and put them in that bag. How many snack bags can she make?*

Sofia: *3*

Zak: When I gave Sofia the word problem about markers, the 4 groups of ten, it was tempting to assume she had an integrated understanding of tens and ones because of the way that she solved that problem. However, when we gave her the other problem, moving the 30 pretzels into bags of 10 each, we got some evidence that maybe she doesn't have quite an integrated understanding with the fact that she drew 10 ones for each one of those and did that 3 times and then needed to count those in order to determine that it would be 3 bags.

Wendy: Right, she's got that understanding that 4 tens is 40 or 3 tens is 30 probably, but thinking about it the opposite way, thinking about it as how many tens are in that 30, she just wasn't quite there yet. She needed to count it out.

Zak: That's right.

Wendy: So, it just shows us how complex thinking about these different ways of thinking in tens and ones and how complex it is for kids and how it really just takes a while to develop that integrated understanding.